XXII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
The correspondence round

Below is the list of problems for the first (correspondence) round of the XXII Sharygin Geometrical Olympiad.

The olympiad is intended for high-school students of four eldest grades. In Russian school, these are 8-11. In the list below, each problem is indicated by the numbers of Russian school grades, for which it is intended. Foreign students of the last grade (at the start of the correspondence round) have to solve the problems for 11th grade, students of the preceding grade solve the problems for 10th grade etc. However, the participants may solve problems for elder grades as well (solutions of problems for younger grades will not be considered).

A complete solution of each problem costs 7 points. A partial solution costs from 1 to 6 points. A text without significant advancement costs 0 points. The result of a participant is the sum of all obtained marks.

First write down the statement of the problem, and then the solution. Present your solutions in detail, including all necessary arguments and calculations. Provide all necessary figures of sufficient size. **If a problem has an explicit answer, this answer must be presented distinctly**. Please, be accurate to provide good understanding and correct estimating of your work !

If your solution depends on some well-known theorems from standard textbooks, you may simply refer to them instead of providing their proofs. However, any fact not from the standard curriculum should be either proved or properly referred (with an indication of the source).

You may note the problems which you liked most (this is not obligatory). Your opinion is interesting for the Jury.

The **solutions** for the problems (in Russian or in English) must be **delivered not before December 1, 2025 and not later than on March 1, 2026**. To upload your work, enter the site **https://geomolymp.ru/olympiads**, indicate the language (English) in the right upper part of the page, and follow the instructions.

Attention:

1. The solution of each problem (and of each part of it if any) must be contained in a **separate** pdf, doc, docx or jpg file. If the solution is contained in several files then pack them to an **archive** (zip or rar) and load it.

2. We recommend to prepare the paper using computer or to scan it rather than to photograph it. **In all cases, please check readability of the file before uploading**.

3. If you upload the solution of some problem more than once then only the last version is retained in the checking system. **Thus if you need to change something in your solution then you have to upload the whole solution of the problem again.**

4. After uploading, log in to the server, open the loaded file and check its correctness.

If you have any technical problems with uploading of the work, apply to **geomshar@yandex.ru (DON'T SEND your work to this address)**.

The final round will be held in July–August 2026 in Moscow region. The winners of the correspondence round are invited to it if they don't graduate from school before the round. The cutoff is determined after the examination of the papers of the correspondence round according to the number of participants with any given score. The graduates who are winners of the correspondence round will be awarded by diplomas of the Olympiad. The list of the winners will be published on **www.geometry.ru** up to June 1, 2026. If you want to know your detailed results, please apply to **geomshar@yandex.ru** after publication of the list.

1. (8) Let triangles T and ABC be symmetric about a point P. Points A', B', C' are the reflections of A, B, C about the corresponding sidelines of T. It is known that two of these points coincide. Prove that all three points coincide.

2. (8) An isosceles trapezoid $ABCD$ is circumscribed around a circle touching the lateral side AB at point T. The segments TC and TD meet this circle for the second time at points P, Q respectively. Find all possible values of $TP/PC + TQ/QD$.

3. (8) Let AK be the bisector of a triangle ABC, N be a point on AC such that $\angle NKC = \angle CAB/2$, and L be the midpoint of KN. Prove that $\angle KBN = \angle LAK$.

4. (8) The diagonals of a circumscribed quadrilateral $ABCD$ meet at point X. Prove that there exists a common tangent to the incircles of triangles ABC, BCD, and AXD.

5. (8) Let I be the incenter of a triangle ABC. The perpendicular bisector to AI meets BC at point D; the line AD meets for the second time the circumcircle of ABC at point X. Prove that $|BX - CX| = AX$.

6. (8–9) Let O be the circumcenter of a triangle ABC, I be its incenter, H be the orthocenter, and N be the Nagel point. Prove that $IN = IH$ if and only if ONH is a right angle.

7. (8–9) The side AB of a triangle ABC touches the incircle and the excircle at points P and Q respectively. Let T be the projection of the midpoint of AB to the bisector of angle C. Prove that C, P, Q, T are concyclic.

8. (8–9) Let ABC be a triangle with $\angle B = 30^\circ$, O be the circumcenter of ABC, I be its incenter. The circles AIB, CIB meet BC, AB respectively at points D, E. Prove that D is the orthocenter of triangle OEI.

9. (8–9) Let $ABCD$ be a circumscribed quadrilateral with incenter I. The circles BID and AIC meet at point P, and the rays AB and DC meet at point Q. Let R be the midpoint of PI. Prove that the quadrilateral $ARQD$ is cyclic.

10. (8–9) A circle ω, a point A on it, and a point B are given. Let X be an arbitrary point of ω, and T be the common point of tangents to the circle ABX at X and B. Find the locus of points T.

11. (8–10) Let P and Q lie on the side AC of a triangle ABC in such a way that $PQ = AC/2$. The point B' is the reflection of B about AC. Let D and E be the points on BP and BQ such that the lines AD and CE touch the circles APB' and CQB' respectively. Prove that the circumcircle of triangle BDE touches AC.

12. (8–10) The vertices of a right-angled triangle ABC are points with integer coordinates. Its incircle centered at I touches AB, BC at points C', A' respectively. The lines AA' and CC' meet at point G. Prove that the line IG passes through some point with integer coordinates.

13. (8–11) Let $A_1 \dots A_n$ be a convex polygon. The points $A_1, \dots, A_n$ in some order are vertices of two closed broken lines. What is the maximal possible ratio of their lengths?

14. (9–11) A triangle ABC ($AB < AC$) is given. Let P be a point on the ray BA such that $BP = AC$, and Q be a point on the ray CA such that $CQ = AB$. Let BB_1 and CC_1 be the perpendiculars to the line PQ. Prove that the circles (CB_1Q), (BC_1P) and the external bisector of angle BAC have a common point.

15. (9–11) Prove that the Nagel point of a triangle lies on its incircle if and only if the bisectors of two angles meet the side of the Gergonne triangle cutting the third vertex, at two points such that the segment between them equals a half of this side.

16. (9–11) The line passing through the common point of the diagonals of a trapezoid $ABCD$ and parallel to its bases meets the lateral side AB at point M. Let K be the projection of M to CD. Prove that KM bisects the angle AKB.

17. (9–11) Let O and H be the circumcenter and the orthocenter of an acute-angled triangle ABC. The tangents to the circumcircle of ABC at B and C meet at point T. Points X and Y lie on AB and AC in such a way that $\angle AOX = \angle AOY = \angle BTC$. Prove that $HT \perp XY$.

18. (9–11) A point P inside a triangle ABC is given. The lines BP, CP meet the circle ABC for the second time at points E, F respectively. The circle Ω passes through P, E and meets AC at points B_1, B_2. The lines PB_1, PB_2 meet AB at points C_1, C_2. Prove that C_1, C_2, P, F are concyclic.

19. (10–11) The incircle of a triangle ABC centered at I touches BC, CA, AB at points A', B', C' respectively. Let A_b, A_c, B_a, C_a be the midpoints of segments $A'B$, $A'C$, $B'A$, $C'A$ respectively. The lines A_bB_a and A_cC_a meet at point P. Prove that the reflection of I about P lies on AA'.

20. (10–11) The altitudes AA_1, BB_1, CC_1 of a triangle ABC meet its circumcircle for the second time at points A_2, B_2, C_2 respectively. Let A_3 be the common point of circles ABC and AB_1C_1, distinct from A; the points B_3, C_3 are defined similarly; A_4, B_4, C_4 are the feet of altitudes of triangle $A_1B_1C_1$. Prove that the lines AA_4, BB_4, CC_4, A_2A_3, B_2B_3, C_2C_3 concur.

21. (10–11) Let ABC be a triangle with $\angle A = 2\pi/3$; P be an arbitrary point inside this triangle lying on the bisector of angle A; the lines BP, CP meet AC, AB at points E, F respectively; D be an arbitrary point on the side BC; the lines DE, DF meet PC, PB at points M, N respectivly. Find the value of angle MAN.

22. (10–11) The incircle of a triangle ABC touches BC at point D. Let F be the Feuerbach point, H be the projection of A to DF. Prove that $FH : DF = 1 : 2$.

23. (10–11) A triangle ABC and a point P in the plane are given. The lines AP, BP, CP meet for the second time the circle ABC at points A', B', C' respectively. Find the locus of points Q such that the common points of QA' with BC, QB' with AC, and QC' with AB are collinear.

24. (11) The insphere of a tetrahedron $ABCD$ touches the face ABC at its orthocenter H and touches the faces ABD, ACD at points P, Q respectively. The lines DP, DQ meet the plane ABC at points X, Y. Prove that $\angle XHY = 2\angle XAY$.